Lecture Triangles and trigonometry week 7 consolidation

1. angles of a triangle always sum to 1800

Given a triangle with 2 angles  and find the third angle

Third angle

**Similar triangles**

These triangles are enlargements of each other

1cm

2.5cm

5cm

The larger triangle has the same angles as the smaller triangle. The sides of the larger triangle are all scaled by the same amount from the corresponding sides in the smaller triangle.

What is length of the right hand side of the larger triangle? The larger triangle has sides scaled by a factor of from the smaller triangle. So the right hand side is

**Right angled triangles**

1. Naming sides and labelling sides and angles:

hypotenuse

The right angle is marked with a small square.

The side opposite the right angle is called the hypotenuse

1. Sides are labelled with lower case letters (a, b, c)

and angles with upper case letters (A, B, C)

B

and that is the angle is opposite the side it doesn’t touch

a

c

C

A

b

1. Pythagoras theorem. This only applies in right angled triangles

Consider the triangle in part 4)

That is the square of the longest side is equal to the sum of the squares on the other 2 sides.

m



4

5

y

6

3

Find the missing sides:

so

and

and

so

and

1. **Equilateral triangles** have all equal sides and all equal angles, all angles are
2. **Isosceles triangles** have 2 equal sides and 2 equal angles

z

3

Y

Z=640

By symmetry

1. **Naming sides and labelling sides and angles in a right angled triangle**
   1. See above

The longest side in right angled triangle is called the hypotenuse

* 1. The other sides are named in relation to a marked angle

opposite

adjacent

A

The opposite and adjacent are in relation to angle A

1. **Sine of an angle**

hyp=10

opp=8

opp=4

hyp=5

A

A

These triangles are similar, the angles are the same and the sides of the larger triangle are twice as long. Consider in these triangles. In both triangles, and all similar triangles this will be

This value depends only on the size of A and is called for short

1. This means we can **find the missing sides of a triangle** using of the angle

C

a

b

B

A

If is a right angle and

This tells us the ratio in this triangle

* 1. If we know the length of we can calculate as

Suppose then so

Suppose then so

1. **Example**: given a triangle with and and find

substituting

So

1. and there are 2 more common trigonometric ratios

And

1. **To find the missing side of a triangle** we need to find which trigonometric ratio to use and then apply it.

Suppose is a right angle and length . Find

* 1. First label the sides in relation to , the angle we have:

is the hypotenuse, and is the adjacent.

X=340

hypotenuse y=2.3m

adjacent z

Y

* 1. Decide which ratio to use.

It needs adjacent and hypotenuse so we use

* 1. Now substitute.

This gives (to 3 s.f.)

1. **Inverse of trigonometric ratios**.
   1. is a function

You put in an angle and you get

So if we can put in the function and get

* 1. We can do the inverse of sin. If we know we can find the angle by doing

1. This means we can use trigonometry to **find missing angles**

In a right angled triangle if and find angle .

* 1. In a right angled label the sides we know or want in relation to the angle we know or want .

C

a

opposite b

B

A

adjacent c

* 1. decide which trigonometric ratio

Must be

* 1. substitute

So

to the nearest degree

1. There is a mnemonic: **SOHCAHTOA**

This splits into 3 parts

SOH because

And TOA because

1. **Special triangles**
   1. Consider this isosceles right angle triangle with shortest side 1.

1

The other 2 angles must both be

The length of the hypotenuse can be found from Pythagoras

gives

45

hypotenuse√2

opposite=1

45

adjacent=1

Now find

* 1. Consider this equilateral triangle with side 2.

60

2

2

60

60

2

The angles must all be

The length of the sides must be 2, and half a side must be 1.

Consider half this triangle.

30

2

√3

60

1

This is now a right angle triangle. It has one angle of so the missing angle is .

We can find the height of the triangle using Pythagoras:

gives

Now find

1. **Polar coordinates**

Cartesian coordinates represent a point that is in the direction and in the direction. This can also be represented by an angle and a distance. The angle is measured from the in an anticlockwise direction.

We can convert between cartesian and polar coordinates using trigonometry:

The modulus or length) is given by

The angle is given by so

So cartesian coordinates are equivalent to ( in polar coordinates

Note polar coordinates are always in the order (

1. It is harder to convert to polar coordinates outside the first quadrant

Consider cartesian coordinates

as before.

We want angle

It is easy to calculate angle

But

(-3,2)

r

x

θ

**Radians**

1. In computing it often makes sense to use another angle measure instead of degrees called radians.
2. A radian is the angle needed to move through the length of a radius along the circumference of a circle.

r

r

r

Since the circumference of a circle is there are radians in a whole turn

This means radians

1. We can see this is a table

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| degrees | 360 | 180 | 90 | 45 | 30 | 60 | 1 |  |
| radians |  |  |  |  |  |  |  | 1 |

The values in the table come up often. The final 2 columns give us conversion formulae

So

So

* 1. Find in radians
  2. Find radians in degrees

1. For triangles that do not have a right angle we can use other formulae to find sides and angles
2. **The sine rule** is

or

1. In this triangle find the missing angle
   1. Label the sides and angles

With

C

35=b

76=a

56=A

B

c

* 1. Use sine rule

gives

1. In this triangle find missing length

A=87

c

b=7

B=45

C

a=x

Label the sides and angles

substitute

Gives

1. **The cosine rule**

Look at this triangle, find the missing angle ,

* 1. First label the triangle with where is Label the other sides and angles

With

A=X

b=32

c=29

B

C

a=45

* 1. Use the cosine rule
  2. substitute

1. Look at this triangle, find the missing side

A=55

c=45

b=32

B

C

a=x

* 1. To find the missing side , label the triangle with where we have the angle

Label the other sides and angles

With

* 1. Use the cosine rule
  2. Substitute